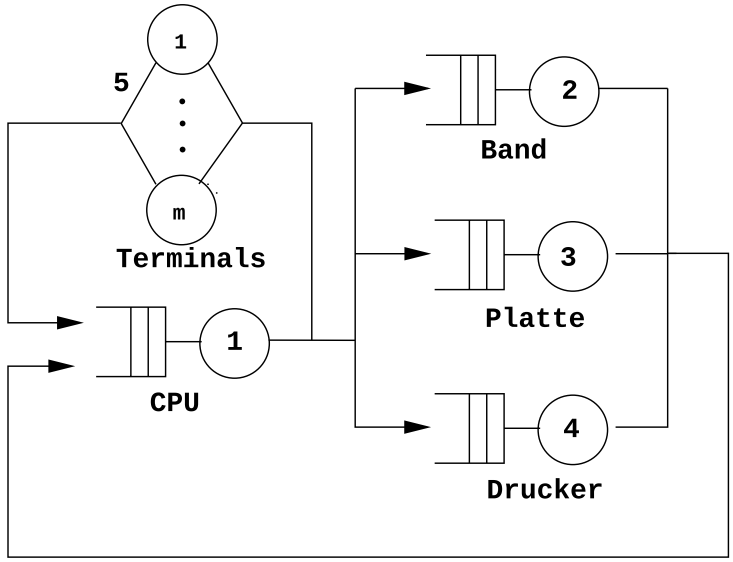
G Applications

■ Terminal System:



◆ System parameters:

- CPU:
  - ➤ Number of processors: 3
  - ➤ Mean service time: 0.5 sec
- Tape:
  - ➤ Mean service time: 5.0 sec
- Disk:
  - ➤ Mean service time: 1.0 sec
- Printer:
  - ➤ Mean service time: 5.0 sec
- Terminals:
  - ➤ Think time: 10 sec
  - ➤ Number: 20

◆ Transition probabilities

$p_{12} = 0.15$

$p_{13} = 0.20$

$p_{14} = 0.15$

$p_{15} = 0.50$

$p_{21} = p_{31} = p_{41} = p_{51} = 1$

◆ Computation of the performance measures using the queueing network tool **PEPSY** (**MVA** or **Convolution)**

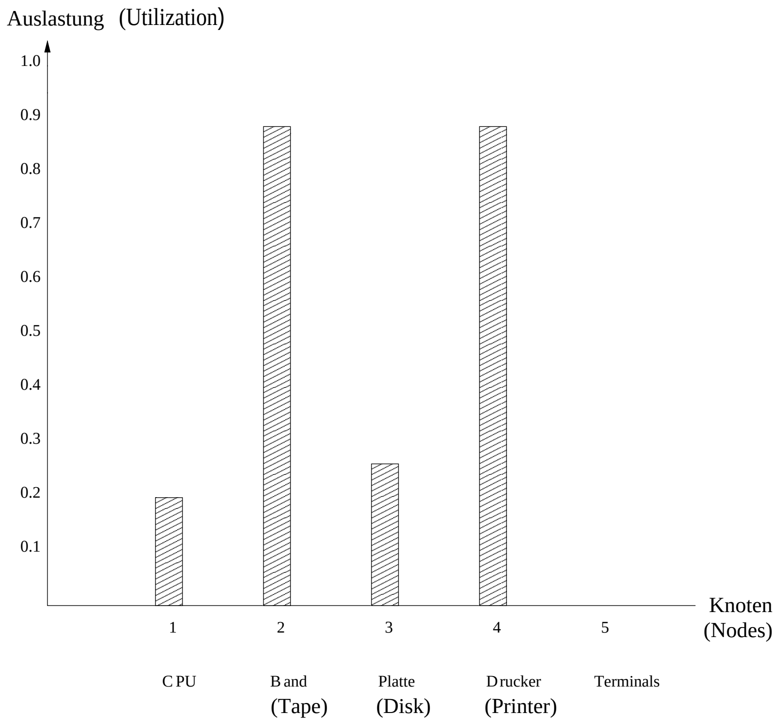
• Performance measures of the individual nodes:

	Service time	Throughput	Utilization	Queue lenght	Response time
CPU	0.500	1.147	**0.191**	0.005	0.504
Tape	5.000	0.172	**0.860**	2.969	22.262
Disk	1.000	0.229	**0.229**	0.066	1.287
Printer	5.000	0.172	**0.860**	2.969	22.262
Terminals	20.000	0.573	--	0.000	20.000

• Performance measures of the network:

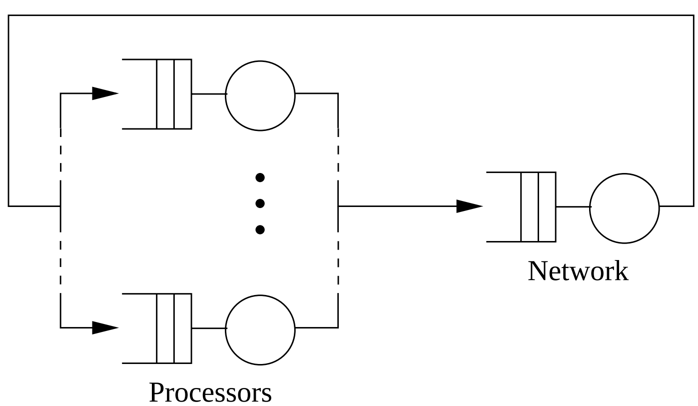
	Throughput	Response time
Network	0.573	34.880

◆ Graphical representation of the results:

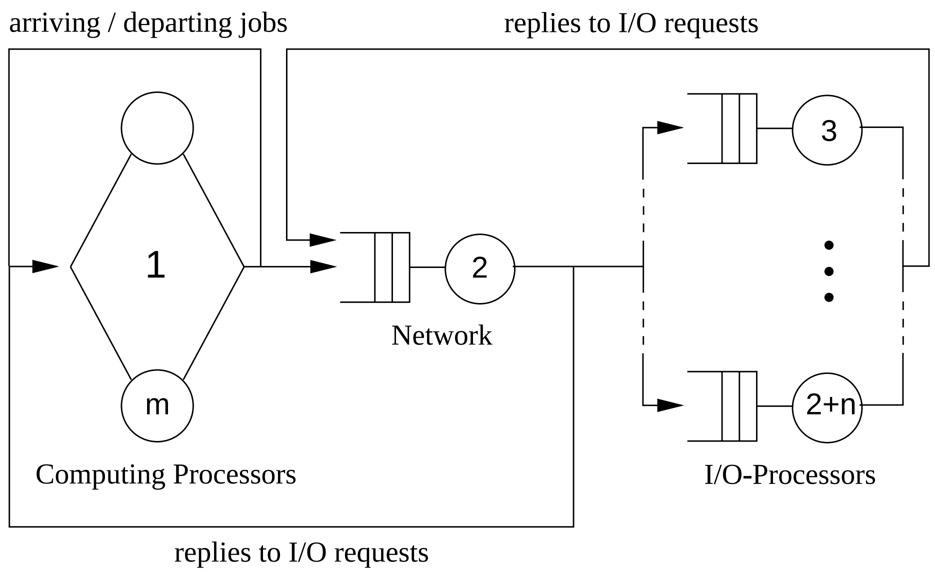


Multiprocessor Systems:

◆ Loosely coupled systems - simple model:



◆ Loosely coupled system - detailed model with I/O-processors:



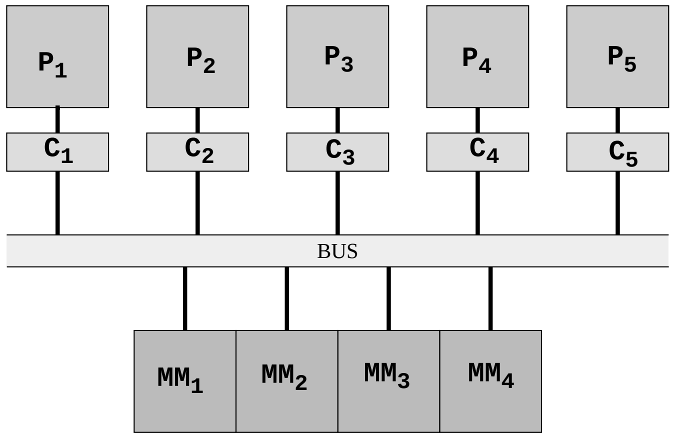
➤ Parameters:

- Number of computing processors $m = 8$
- Number of I/O-processors $n = 2, 3, 4$
- Mean service time of the computing processors $1/\mu_1 = 30\ msec$
- Mean service time of the I/O-processors $1/\mu_i = 50\ msec \quad i = 3, \ldots, n+2$
- Mean respone time of the network $1/\mu_2 = 1\ msec$
- $p_{11} = 0.005; \quad p_{21} = 0.5; \quad p_{2i} = 0.5/n \quad i = 3, \ldots, n+2$

➤ Performance measures:

Number of I/O Processors	2	3	4
mean response time	**4.15 sec**	**3.18 sec**	**2.719 sec**
throughput	**1.93 sec^{-1}**	**2.51 sec^{-1}**	**2.944 sec^{-1}**
$\rho_{\mathbf{computingprocessor}}$	0.145	0.189	0.220
$\rho_{\mathbf{network}}$	0.070	0.090	0.106
$\rho_{\mathbf{I/Oprocessor}}$	0.867	0.754	0.662

◆ Multiprocessor System - tightly coupled systems:

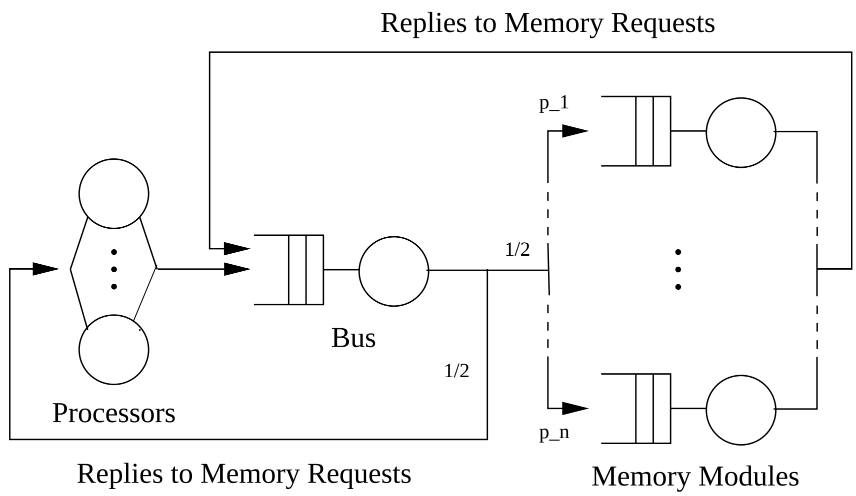


P_n	Processor n
C_n	Cache n
MMn	Memory Modul n

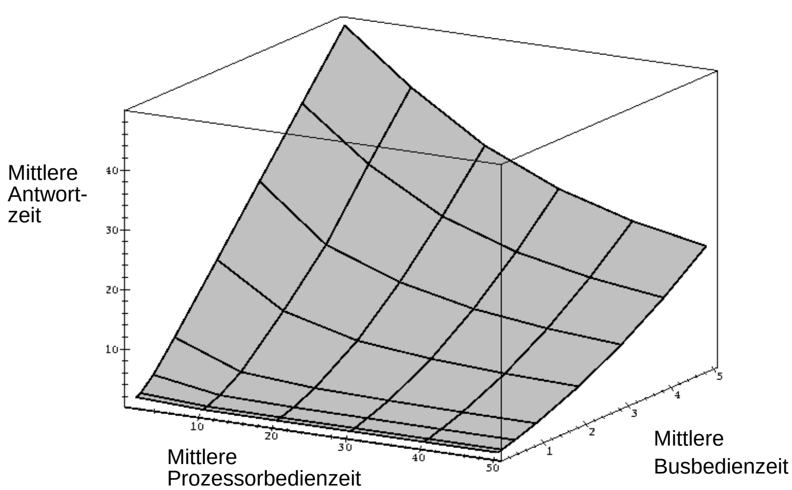
◆ System parameters:

- ➤ Mean bus service time:
  Mean time the bus is occupied by a request
- ➤ Mean memory service time:
  Mean time for a memory request
- ➤ Mean time between two successive memory requests (cache misses)
- ➤ Probability p_n of a request to memory modul n

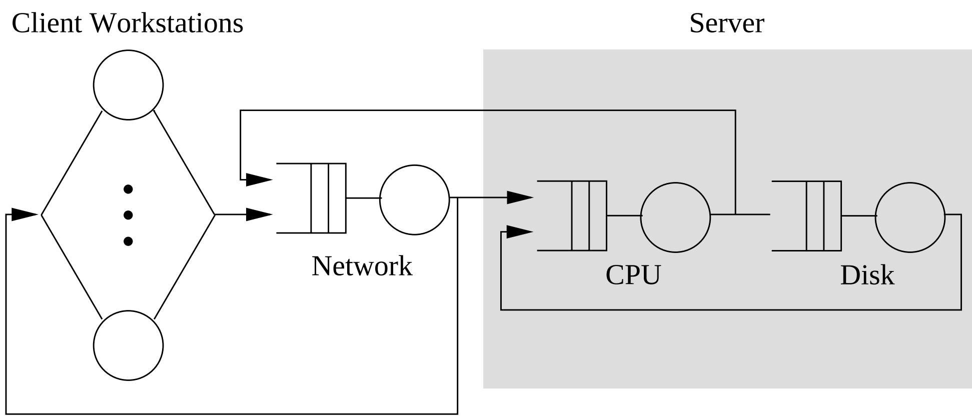
◆ Queueing network model:



◆ Mean response time of a memory request of a processor:



Client-Server System:



Network: Ethernet (CSMA/CD)

Parameter	Description
N_p	Average number of packets generated per request
B	Network bandwidth in bits per second
S	Slot duration (i.e., time for collision detection)
$\overline{L}_p$	Average packet length in bits

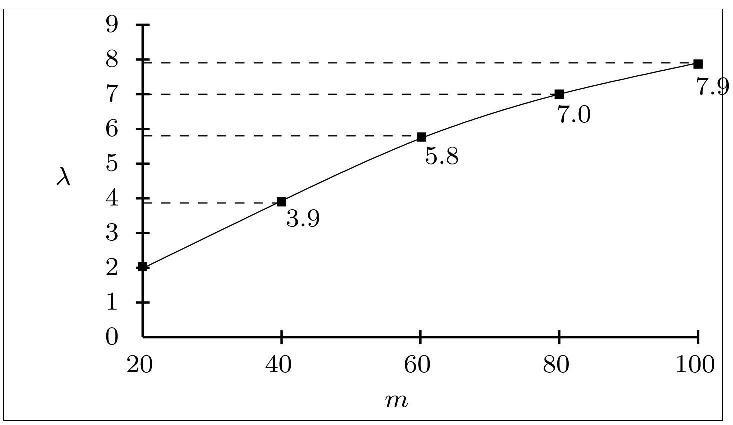
$$\mu_{\text{net}}(k) = \begin{cases} \left(\frac{1}{N_p} \cdot \frac{\overline{L}_p}{B} + S \cdot C(1)\right)^{-1}, & k = 1\,, \\ \left(\frac{1}{N_p} \cdot \frac{\overline{L}_p}{B} + S \cdot C(k+1)\right)^{-1}, & k > 1\,, \end{cases}$$

$$C(k) = \big(1 - A(k)\big)/A(k)$$
$$A(k) = (1 - 1/k)^{k-1}$$

k: Number of workstations that desire use of the network

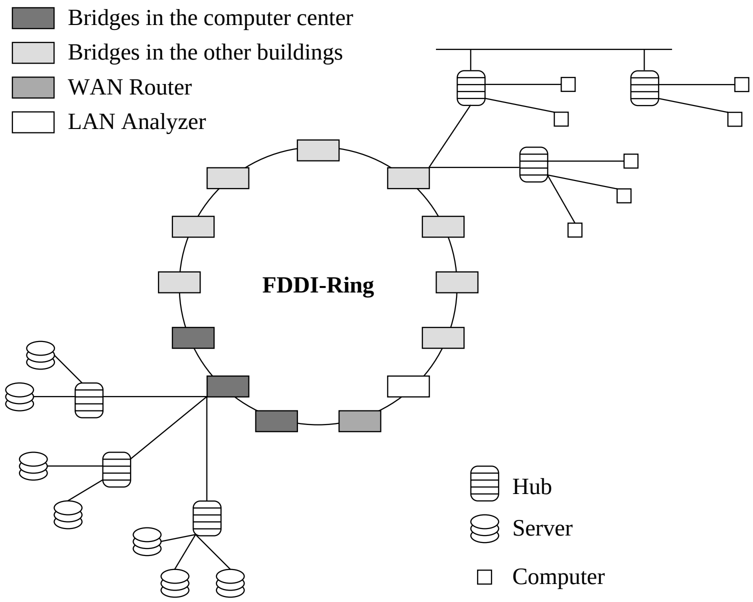
$N_p = 7$		$\mu_1 = \mu_{CL} = 0.1/\sec$	
$B = 10\ \mathrm{Mb}/\sec$		$\mu_2 = \mu_{Net}(k)$	
$S = 51.2\ \mu\sec$		$\mu_3 = \mu_{CPU} = 16.7/\sec$	
$\overline{L}_p = 1518$ bits		$\mu_4 = \mu_{Disk} = 18.5/\sec$	
$p_{12} = 1$	$p_{21} = 0.5$	$p_{32} = 0.5$	$p_{43} = 1$
	$p_{23} = 0.5$	$p_{34} = 0.5$	

Throughput

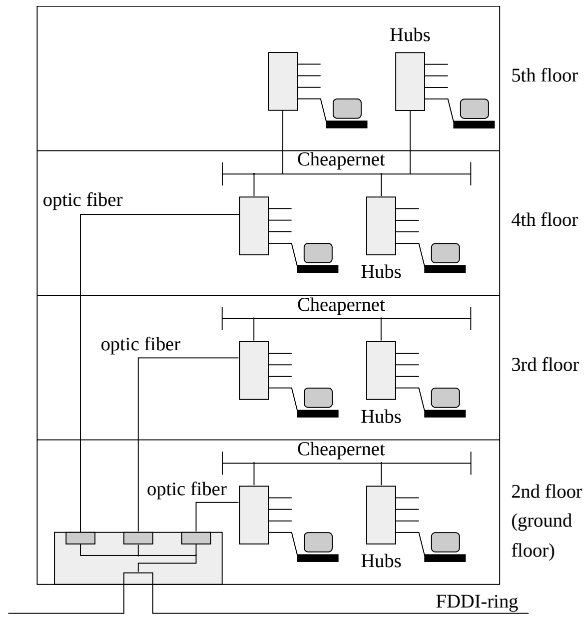


Number of Workstations

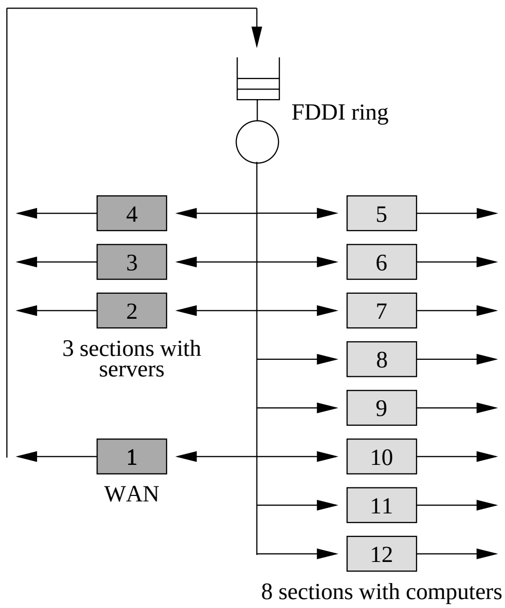
Communikation System with FDDI-Ring and Ethernets:



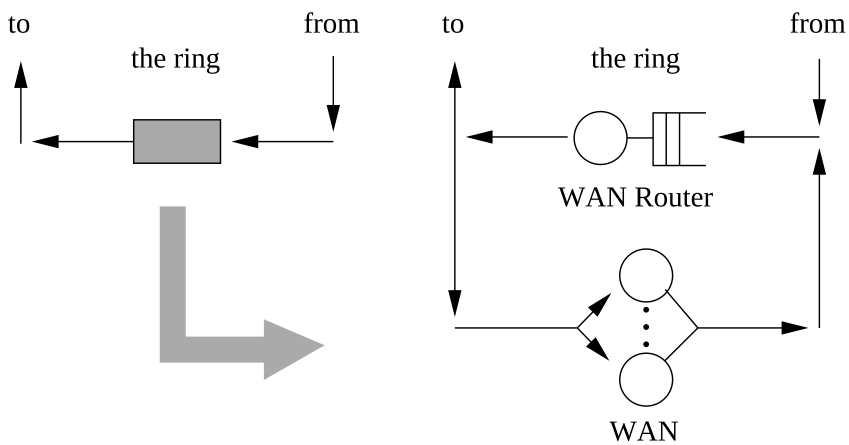
◆ Ethernet in a building:



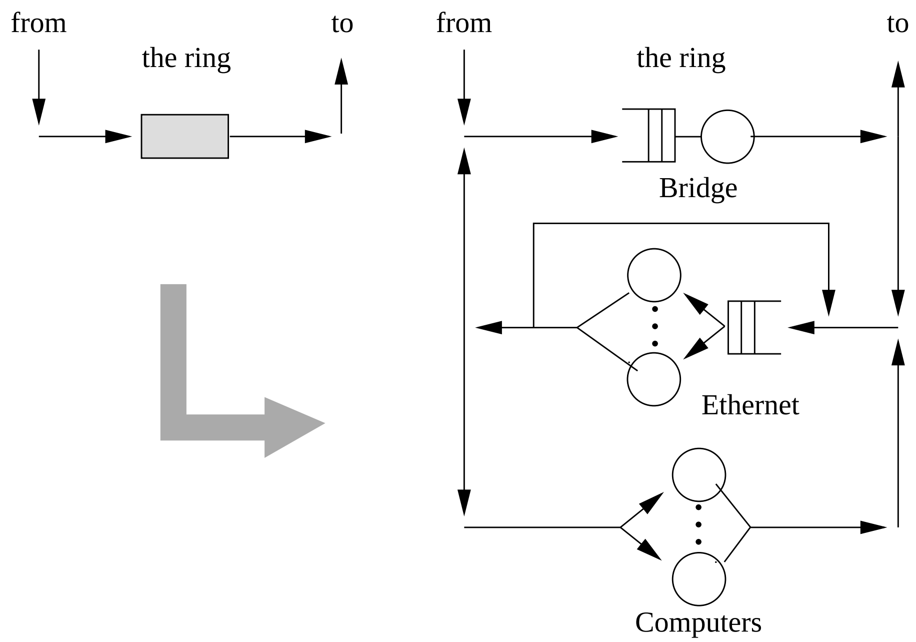
◆ **Closed queueing network model of the LAN with a simplified representation of the individual sections:**



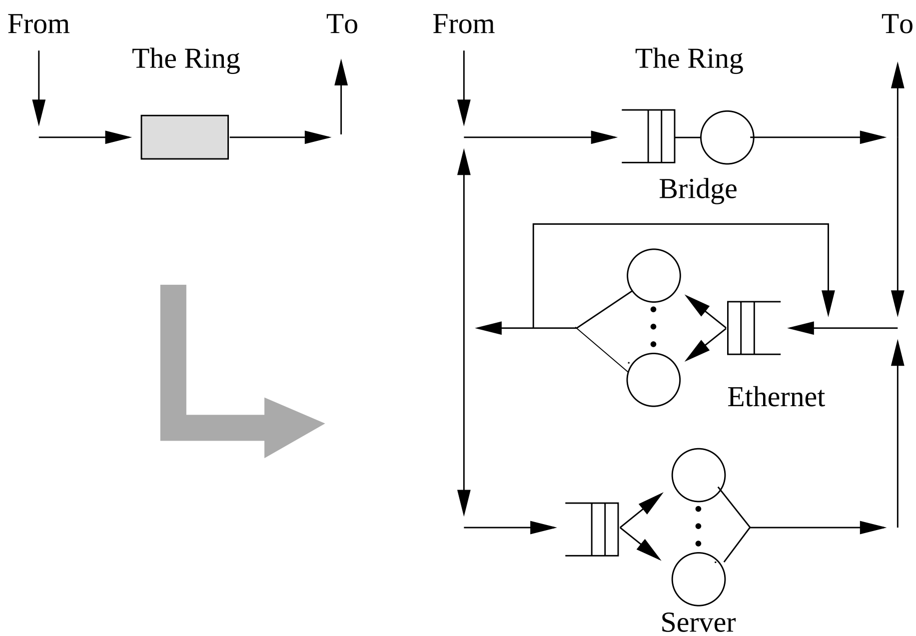
◆ **Model of the WAN and the WAN-Router:**



◆ **Model of a section of the LAN with computers:**



◆ **Model of a section of the LAN with servers:**



◆ Determination of the system parameters:

➤ Total data transferred from FDDI-Ring to the bridges for a two day period Tage and estimated routing probabilities:

Section	1	2	3	4	5	6	7	8	9	10	11	12
Data/Mb	2655	1690	2800	1652	2840	1500	3000	200	1940	1180	4360	4380
$p_i/\%$	9.5	6.2	10	5.9	10.1	5.4	10.7	0.7	6.9	4.2	14.8	15.6

1: WAN; 2, 3, 4: sections with server; 5, ... , 12: sections with computers

➤ Collision probabilities:

Section	1	2	3	4	5	6	7	8	9	10	11	12
$q_i/\%$	1	5	1	1	1	1	1	1	1	1	3	2

➤ Number of service units in nodes *i*:

Section	**2**	**3**	**4**
m_i	**14**	**23**	**13**

Section	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
m_i	**4**	**7**	**2**	**4**	**5**	**3**	**1**	**4**	**2**	**5**	**5**

➤ Measured values of the distribution of the inter arrival times and the packet length:

Interarrival Time (μs)	%
$\leq$ 5	**3.0**
5 – 20	**0.9**
20 – 82	**15.3**
82 – 328	**42.7**
329 – 1300	**27.1**
1300 – 5200	**1.0**

Length (Bytes)	%
$\leq$ 32	**11.1**
32 – 63	**10.0**
64 – 95	**33.0**
96 – 127	**7.9**
128 – 191	**6.8**
192 – 511	**4.1**
512 – 1023	**5.8**
1024 – 1526	**21.3**

➤ Mean value and coefficient of variation of the inter arrival time and the packet length:

	Interarrival Times of the Frames	Length of the Frames
Mean value	**346 μsec**	**382.5** Byte
Squared coefficient of variation	**1.48**	**1.67**

➤ Arrival rate of the packets at the Sections (using the routing probabilities and $\lambda = 1/(346\mu sec) = 2890/sec$):

Station	1	2	3	4	5	6	7	8	9	10	11	12
λ_i	275	179	289	171	292	156	309	20	199	121	428	451

➤ Mean token rotation tiome:

$$\overline{T}_r = U + R^{-1} \cdot \overline{L} \cdot \sum \rho_i$$

- $U = 22\ \mu sec$: Mean rotation time of a free token
- $R = 100\ Mb/sec$: Transfer rate
- $\overline{L}$: Mean packet length $\rightarrow$ Table

$$\sum \rho_i = \frac{\sum \lambda_i}{\mu} = \frac{\lambda}{\mu}$$

Approximation:

$$\overline{T}_r \approx \frac{1}{\mu}$$

➤ Service rate of the token ring:

$$\mu = \frac{R - \lambda \overline{L}}{U \cdot R}$$

- $\lambda = 1/(346\mu\text{sec}) = 2890/sec$: Throughput

$$\mu = 41435/\sec \qquad \overline{T}_r \approx \frac{1}{\mu} = 24\ \mu\text{sec}$$

➤ Variance of the token rotation time:

$$\sigma^2_{T_r} = R^{-2}\left(\rho \cdot \operatorname{var}(L) + \rho \cdot \left(1 - \rho \sum p_i^2\right) \overline{L}^2\right)$$

$\rho = \lambda/\mu$; variance and 2. moment of L $\rightarrow$ Table

$$c^2_{T_r} = 0.3$$

- Service times at the "bridges" is constant ($c_{Bridge} = 0$) with the service rate $\mu_{Bridge} = 10000$ packets/sec

- Service time of the ethernet T_{eth} :

$$T_{eth} = T_t + T_d$$

- T_t: Transfer time = mean packet length/transfer rate

  packet length (from table, with minimal packet length 72 bytes in the ethernet):

$$\overline{L}_{eth} = 395 \text{ bytes} = 3160 \text{ bit}, \quad c^2_{L_{eth}} = 1.51 .$$

Transfer rate: 10 Mb/sec

$$\overline{T}_t = \frac{3160 \text{ bit}}{10 \text{ Mb/sec}} = 316\ \mu\text{sec} \quad c^2_{T_t} = 1.51 .$$

- Mean delay time of the ethernet T_d, calculated by the characteristics of the ethernets:

	Transfer Rate	Mean Length	Signal Time
Optical fiber	10 Mb/sec	11 m	5 μs ec/km
Cheapernet	10 Mb/sec	5 m	4.3 μsec /km
Twisted pair	10 Mb/s	50 m	4.8 μse c/km

$$\begin{aligned}\overline{T}_d &= 0.011 \text{ km} \cdot 5\ \mu\text{sec/km} + 0.005 \text{ km} \cdot 4.3\ \mu\text{sec/km} \\ &+ 0.05 \text{ km} \cdot 4.8\ \mu\text{sec/km} \\ &= 0.3\ \mu\text{sec} .\end{aligned}$$

- Mean delay time of the ethernet T_d can be neglected compared to the transfer time T_t and we obtain for the mean service time of the ethernets:

$$\overline{T}_{eth} = 1/\mu_{eth} = 316\ \mu\text{sec}$$

➤ Approximation of the service time of the WANs and of the computers of a section (= IS-node):

$$\overline{K}_i = \frac{\lambda_i}{\mu_i}, \qquad \rightarrow \qquad \mu_i = \frac{\lambda_i}{\overline{K}_i}.$$

with $K = 170$: mean number of active computers. = mean number of jobs in the network:

$$\overline{K}_i \approx \frac{170}{12} = 14.17$$

➤ Approximation of the service time of the sections with servers = -/G/m-nodes

$$\rho_i = \frac{\lambda_i}{m_i \mu_i}, \qquad \rightarrow \qquad \mu_i = \frac{\lambda_i}{m_i \rho_i},$$

Utilization of the section with servers estimated to 90%

➤ Service rates of the WANs and sections with computers respectively Servern:

Station	1	2	3	4	5	6	7	8	9	10	11	12
μ_i	19.4	14.2	14.0	14.6	20.6	11.0	21.8	14.1	14.8	8.5	30. 2	31.8

◆ Applicatiopn of the queueing network tool **PEPSY** to analyze the system:

➤ Closed non-product-form queueing network

➤ Method of Marie

➤ 36 nodes

➤ $K = 170$

```
#
# filename  e_lan170
#

NUMBER NODES: 36
NUMBER CLASSES: 1

NODE SPECIFICATION

  node |  name          |     type       | node |  name          |     type
-------+----------------+----------------+------+----------------+--------------
   1   |  ring          |  -/G/1-FCFS    |  19  |  pc-b7         |  -/G/0-IS
   2   |  bridge-cc2    |  -/G/1-FCFS    |  20  |  bridge-b8     |  -/G/1-FCFS
   3   |  eth-cc2       |  -/G/4-FCFS    |  21  |  eth-b8        |  -/G/1-FCFS
   4   |  serv-cc2      |  -/G/14-FCFS   |  22  |  pc-b8         |  -/G/0-IS
   5   |  bridge-cc3    |  -/G/1-FCFS    |  23  |  bridge-b9     |  -/G/1-FCFS
   6   |  eth-cc3       |  -/G/7-FCFS    |  24  |  eth-b9        |  -/G/4-FCFS
   7   |  serv-cc3      |  -/G/23-FCFS   |  25  |  pc-b9         |  -/G/0-IS
   8   |  bridge-cc4    |  -/G/1-FCFS    |  26  |  bridge-b10    |  -/G/1-FCFS
   9   |  eth-cc4       |  -/G/2-FCFS    |  27  |  eth-b10       |  -/G/2-FCFS
  10   |  serv-cc4      |  -/G/13-FCFS   |  28  |  pc-b10        |  -/G/0-IS
  11   |  bridge-b5     |  -/G/1-FCFS    |  29  |  bridge-b11    |  -/G/1-FCFS
  12   |  eth-b5        |  -/G/4-FCFS    |  30  |  eth-b11       |  -/G/5-FCFS
  13   |  pc-b5         |  -/G/0-IS      |  31  |  pc-b11        |  -/G/0-IS
  14   |  bridge-b6     |  -/G/1-FCFS    |  32  |  bridge-b12    |  -/G/1-FCFS
  15   |  eth-b6        |  -/G/5-FCFS    |  33  |  eth-b12       |  -/G/5-FCFS
  16   |  pc-b6         |  -/G/0-IS      |  34  |  pc-b12        |  -/G/0-IS
  17   |  bridge-b7     |  -/G/1-FCFS    |  35  |  wanrouter     |  -/G/1-FCFS
  18   |  eth-b7        |  -/G/3-FCFS    |  36  |  wan           |  -/G/0-IS

CLASS SPECIFICATION

  class    | arrival rate   number of jobs
----------+---------------------------------
     1    |     -            170
```



```
#
# filename  e_lan170
#

NUMBER NODES: 36
NUMBER CLASSES: 1


CLASS SPECIFIC PARAMETERS

CLASS  1                  ( sc_o_v = squared coeffcent of variation )

 node        | service_rate | sc_o_v | visit_rat | node        | service_rate | sc_o_v | visit_rat
-------------+--------------+--------+-----------+-------------+--------------+--------+----------
 ring        |   41345      |   0.3  | 9.901     | pc-b7       |  21.81       |  1     | 1.059
 bridge-cc2  |   9999       |   0.1  | 1.228     | bridge-b8   |  9999        |  0.1   | 0.139
 eth-cc2     |   3164       |   1.51 | 1.292     | eth-b8      |  3164        |  1.5   | 0.14
 serv-cc2    |   14.2       |   1    | 0.614     | pc-b8       |  14.11       |  1     | 0.069
 bridge-cc3  |   9999       |   0.1  | 1.98      | bridge-b9   |  9999        |  0.1   | 1.366
 eth-cc3     |   3164       |   1.51 | 2         | eth-b9      |  3164        |  1.51  | 1.38
 serv-cc3    |   14.0       |   1    | 0.99      | pc-b9       |  14.11       |  1     | 0.683
 bridge-cc4  |   9999       |   0.1  | 1.168     | bridge-b10  |  9999        |  0.1   | 0.832
 eth-cc4     |   3164       |   1.51 | 1.18      | eth-b10     |  3164        |  1.51  | 0.84
 serv-cc4    |   14.6       |   1    | 0.584     | pc-b10      |  8.54        |  1     | 0.416
 bridge-b5   |   9999       |   0.1  | 2         | bridge-b11  |  9999        |  0.1   | 2.93
 eth-b5      |   3164       |   1.51 | 2.02      | eth-b11     |  3164        |  1.51  | 3.021
 pc-b5       |   20.61      |   1    | 1         | pc-b11      |  30.21       |  1     | 1.465
 bridge-b6   |   9999       |   0.1  | 1.069     | bridge-b12  |  9999        |  0.1   | 3.089
 eth-b6      |   3164       |   1.51 | 1.08      | eth-b12     |  3164        |  1.51  | 3.152
 pc-b6       |   11.01      |   1    | 0.535     | pc-b12      |  31.83       |  1     | 1.545
 bridge-b7   |   9999       |   0.1  | 2.119     | wanrouter   |  9999        |  0.1   | 1.881
 eth-b7      |   3164       |   1.51 | 2.1       | wan         |  19.41       |  1     | 0.941
```

PERFORMANCE_MEASURE FOR NETWORK: lan170

description of the network is in file 'e_lan170'

the closed network was solved using the method 'marie'

jobclass 1

marie	lambda	e	1/mu	rho	mvz	maa	mwz	mwsl
ring	2879.034	9.901	0.000	0.070	0.000	0.377	0.000	0.089
bridge-cc2	357.080	1.228	0.000	0.036	0.000	0.037	0.000	0.001
eth-cc2	375.690	1.292	0.000	0.030	0.000	0.119	0.000	0.000
serv-cc2	178.540	0.614	0.070	0.898	0.088	15.669	0.017	3.096
bridge-cc3	575.749	1.980	0.000	0.058	0.000	0.060	0.000	0.002
eth-cc3	581.564	2.000	0.000	0.026	0.000	0.184	0.000	0.000
serv-cc3	287.874	0.990	0.071	0.894	0.078	22.479	0.007	1.916
bridge-cc4	339.634	1.168	0.000	0.034	0.000	0.035	0.000	0.001
eth-cc4	343.123	1.180	0.000	0.054	0.000	0.113	0.000	0.004
serv-cc4	169.817	0.584	0.069	0.895	0.087	14.858	0.019	3.227
bridge-b5	581.564	2.000	0.000	0.058	0.000	0.060	0.000	0.002
eth-b5	587.380	2.020	0.000	0.046	0.000	0.186	0.000	0.000
pc-b5	290.782	1.000	0.049	0.000	0.049	14.109	0.000	0.000
bridge-b6	310.846	1.069	0.000	0.031	0.000	0.032	0.000	0.001
eth-b6	314.045	1.080	0.000	0.020	0.000	0.099	0.000	0.000
pc-b6	155.568	0.535	0.091	0.000	0.091	14.130	0.000	0.000
bridge-b7	616.167	2.119	0.000	0.062	0.000	0.064	0.000	0.003
eth-b7	622.274	2.140	0.000	0.066	0.000	0.199	0.000	0.003
pc-b7	307.938	1.059	0.046	0.000	0.046	14.119	0.000	0.000

characteristic indices:

marie	lambda	mvz	maa
	290.782	0.585	170.000

PERFORMANCE_MEASURE FOR NETWORK: lan170

description of the network is in file 'e_lan170'

the closed network was solved using the method 'marie'

jobclass 1

marie	lambda	e	1/mu	rho	mvz	maa	mwz	mwsl
bridge-b8	40.419	0.139	0.000	0.004	0.000	0.004	0.000	0.000
eth-b8	40.709	0.140	0.000	0.013	0.000	0.013	0.000	0.000
pc-b8	20.064	0.069	0.071	0.000	0.071	1.422	0.000	0.000
bridge-b9	397.208	1.366	0.000	0.040	0.000	0.041	0.000	0.001
eth-b9	401.279	1.380	0.000	0.032	0.000	0.127	0.000	0.000
pc-b9	198.604	0.683	0.071	0.000	0.071	14.075	0.000	0.000
bridge-b10	241.931	0.832	0.000	0.024	0.000	0.025	0.000	0.000
eth-b10	244.257	0.840	0.000	0.039	0.000	0.078	0.000	0.001
pc-b10	120.965	0.416	0.117	0.000	0.117	14.165	0.000	0.000
bridge-b11	852.282	2.931	0.000	0.085	0.000	0.090	0.000	0.005
eth-b11	878.453	3.021	0.000	0.056	0.000	0.278	0.000	0.000
pc-b11	425.996	1.465	0.033	0.000	0.033	14.101	0.000	0.000
bridge-b12	898.226	3.089	0.000	0.090	0.000	0.096	0.000	0.006
eth-b12	916.545	3.152	0.000	0.058	0.000	0.290	0.000	0.000
pc-b12	449.258	1.545	0.031	0.000	0.031	14.114	0.000	0.000
wanrouter	546.961	1.881	0.000	0.055	0.000	0.057	0.000	0.002
wan	273.626	0.941	0.051	0.000	0.051	14.097	0.000	0.000

characteristic indices:

marie	lambda	mvz	maa
	290.782	0.585	170.000

➤ Variation of the number of active computers = number of packets in the network:

K	100	130	150	170	180	200	300
ρ_2	0.55	0.71	0.81	0.90	0.93	0.97	0.99
ρ_3	0.55	0.71	0.81	0.89	0.92	0.97	0.99
ρ_4	0.55	0.71	0.81	0.90	0.92	0.97	0.99
$\overline{Q}_2$	0.05	0.6	1.4	3.1	4.8	7.9	50
$\overline{Q}_3$	0.02	0.3	0.8	1.9	2.9	8.3	30
$\overline{Q}_4$	0.11	0.6	1.4	3.2	4.8	9.3	43
$\overline{T}$	0.56	0.56	0.57	0.59	0.60	0.64	0.94
ρ_{ring}	0.043	0.055	0.063	0.070	0.072	0.075	0.077

$m_2 = 14; \quad m_3 = 23; \quad m_4 = 13$

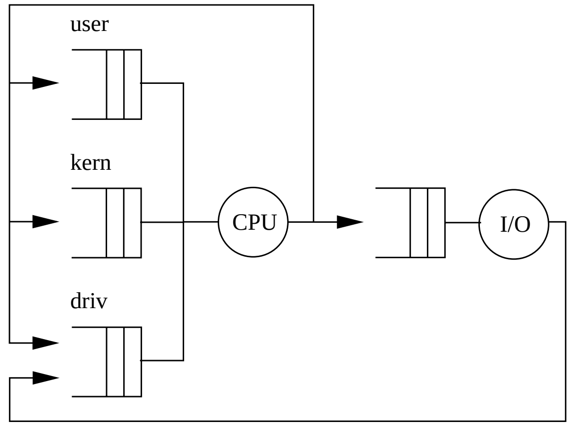
➤ Variation of the number of servers:

m_2	7	10	14	16	18	28
m_3	12	16	23	18	26	46
m_4	7	9	13	16	17	26
ρ_2	0.999	0.96	0.90	0.69	0.73	0.47
ρ_3	0.95	0.98	0.89	0.998	0.88	0.47
ρ_4	0.92	0.98	0.90	0.64	0.71	0.47
$\overline{Q}_2$	63	10	3.1	0.4	0.5	0
$\overline{Q}_3$	9.6	13	1.9	28	0.7	0
$\overline{Q}_4$	7.0	23	3.2	0.2	0.5	0
$\overline{T}$	1.05	0.77	0.59	0.67	0.56	0.56
ρ_{ring}	0.039	0.053	0.070	0.061	0.072	0.073

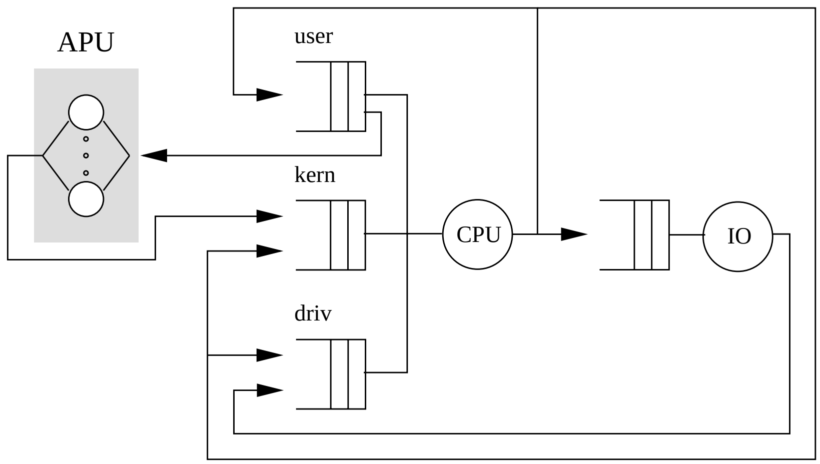
$K = 170$

Model of a UNIX-Kernel:

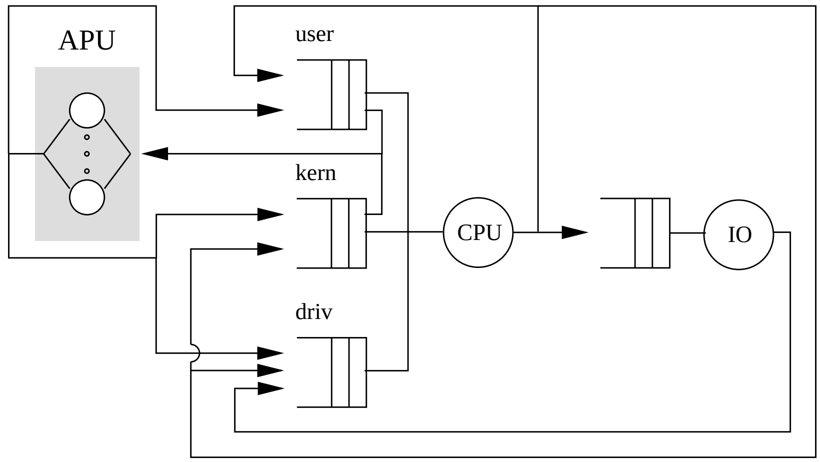
◆ **Monoprocessor**:



◆ **Multiprocessor** (Master-Slave-Configuration)**:**



◆ **Multiprocessor** (extended Master-Slave-Configuration):



◆ System description:

➤ Closed queueing network with K = 10 Jobs

➤ Exp. distr. service times

➤ Priority classes *user, kern, driv* Priority order: *driv* > *kern* > *user*

➤ Class switching is allowed

➤ *user*-jobs can be preempted by *kern*- and *driv*-jobs.
No other preemptions are possible.

➤ Transition probabilities and mean service times:

$$p_{\mathbf{io}} = \mathbf{0.05}, \quad s_{\mathbf{user}} = \text{varied from } \mathbf{0.25} \text{ to } \mathbf{20.0},$$
$$p_{\mathbf{done}} = \mathbf{0.01/0.005}, \quad s_{\mathbf{kern}} = \mathbf{1.0},$$
$$p_{\mathbf{drivdone}} = \mathbf{0.4}, \quad s_{\mathbf{driv}} = \mathbf{0.5}.$$

➤ Transition probabilities:

	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
(1,1)	0	p_{drivdone}	0	$1 - p_{\text{drivdone}}$	0	0
(1,2)	p_{io}	p_{done}	p_{user}	0	0	0
(1,3)	0	1	0	0	0	0
(2,1)	1	0	0	0	0	0
(2,2)	0	0	0	0	0	0
(2,3)	0	0	0	0	0	0

```
node 1: CPU        class 1: driv
node 2: I/O        class 2: kern
                   class 3: user
```

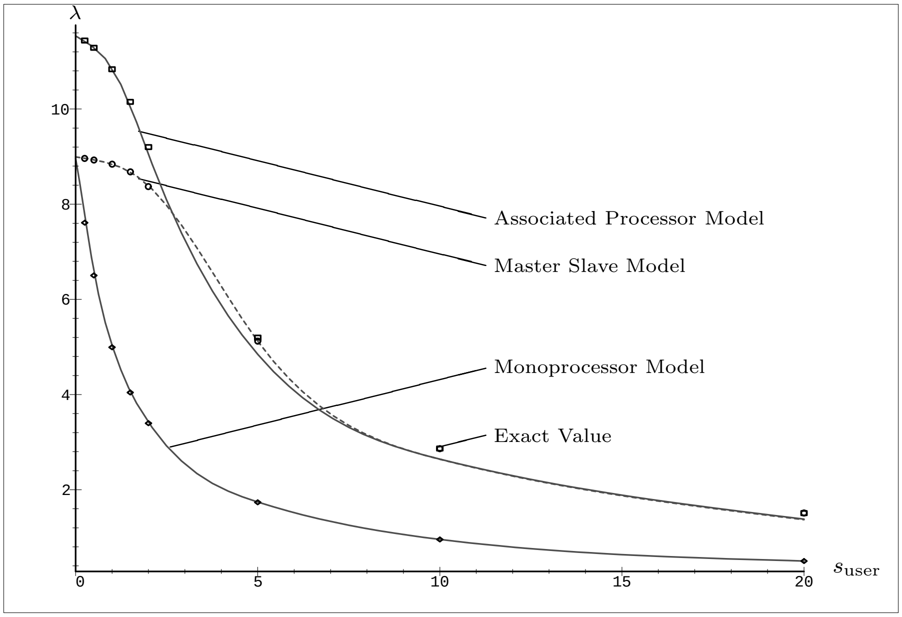

➤ The I/O-system consists of several devices which are combined to a "load dependent node" with the following measured mean service times:

$$
\begin{aligned}
s_{\text{io}}(1) &= 28.00, & s_{\text{io}}(6) &= 12.444,\\
s_{\text{io}}(2) &= 18.667, & s_{\text{io}}(7) &= 12.000,\\
s_{\text{io}}(3) &= 15.555, & s_{\text{io}}(8) &= 11.667,\\
s_{\text{io}}(4) &= 14.000, & s_{\text{io}}(9) &= 11.407,\\
s_{\text{io}}(5) &= 13.067, & s_{\text{io}}(10) &= 11.200.
\end{aligned}
$$

◆ Queueing network → Priority network with mixed priorities (with and without preemption) and class switching.

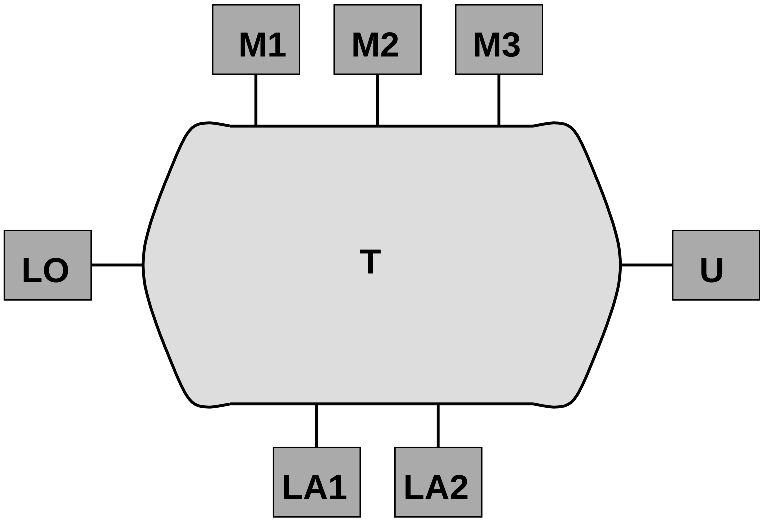
◆ Solution: Extension of the Shadow Method.

◆ Throughput as a function of the *user*-service time for the 3 models: :

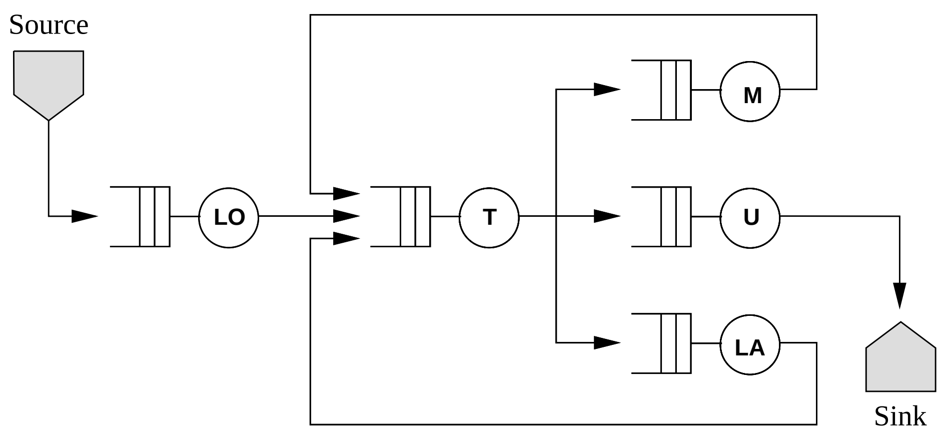


Flexible Production System:

◆ System model:



◆ Queueing network model:



◆ System description:

- ➤ LO: Load station where the work pieces are mounted to the pallet
- ➤ LA1 und LA2: Two identical lathes
- ➤ M1, M2 und M3: Threeidentical milling machines
- ➤ T: Tranfer system that does the transfer between the stations and consists of two automatically controled vehicles.
- ➤ U: A station to unload the pallet, which removes the work pieces from the system
- ➤ Probabilities q_{ij} $i = LO, M, LA; j = M, LA, U$ that the transfer system T moves work pieces from staion i to station j .

$i \backslash j$	M	LA	U
LO	0.5	0.5	0
M	0	0.4	0.6
LA	0.7	0	0.3

➤ Transition probabilities of the queueing network model p_{ij} $i, j = LO, LA, M, U, T$:

$$p_{T,M} = \frac{\lambda_{LO}}{\lambda_T} \cdot q_{LO,M} + \frac{\lambda_{LA}}{\lambda_T} \cdot q_{LA,M} \,,$$
$$p_{T,LA} = \frac{\lambda_{LO}}{\lambda_T} \cdot q_{LO,LA} + \frac{\lambda_M}{\lambda_T} \cdot q_{M,LA} \,,$$
$$p_{T,U} = \frac{\lambda_{LA}}{\lambda_T} \cdot q_{LA,U} + \frac{\lambda_M}{\lambda_T} \cdot q_{M,U} \,.$$

$$p_{T,M} = \frac{1}{\lambda_T}\left(\lambda_{LO} \cdot 0.5 + \lambda_{LA} \cdot 0.7\right) \,,$$
$$p_{T,LA} = \frac{1}{\lambda_T}\left(\lambda_{LO} \cdot 0.5 + \lambda_M \cdot 0.4\right) \,,$$
$$p_{T,U} = \frac{1}{\lambda_T}\left(\lambda_{LA} \cdot 0.3 + \lambda_M \cdot 0.6\right) \,,$$

➤ Application of Jackson’s Theorem for open networks:

– Service times and inter arrival times exp. distr.

– Strategy FCFS

– System parameters (time unit: 1 hour):

i	λ_{0i}	μ_i	m_i
LO	15	20	1
LA	0	10	2
M	0	7	3
U	0	20	1
T	0	24	2

➤ Computation of the throughputs using the traffic equations:

$$\lambda_i = \lambda_{0i} + \sum_j \lambda_j \cdot p_{ji}\,, \quad i,j = LO, LA, M, U, T$$

$$\begin{aligned}
\lambda_{LO} &= \lambda_{0\,LO} = 15\,,\\
\lambda_{LA} &= \lambda_T \cdot p_{T,LA} = \lambda_{LO} \cdot 0.5 + \lambda_M \cdot 0.4 = 14.58\,,\\
\lambda_M &= \lambda_T \cdot p_{T,M} = \lambda_{LO} \cdot 0.5 + \lambda_{LA} \cdot 0.7 = 17.71\,,\\
\lambda_U &= \lambda_T \cdot p_{T,U} = \lambda_{LO} = 15\,,\\
\lambda_T &= \lambda_{LO} + \lambda_{LA} + \lambda_M = 47.29\,.
\end{aligned}$$

➤ Performance measures of the individual nodes:

i	$\overline{Q}_i$	$\overline{W}_i$	ρ_i
LO	2.25	0.15	0.75
LA	1.66	0.11	0.73
M	3.86	0.22	0.84
U	2.25	0.15	0.75
T	65.02	1.38	0.985

Addition of a third vehicle in the transfer system:

$$\rho_T = 0.66\,, \quad \overline{Q}_T = 0.82\,, \quad \overline{W}_T = 0.02$$

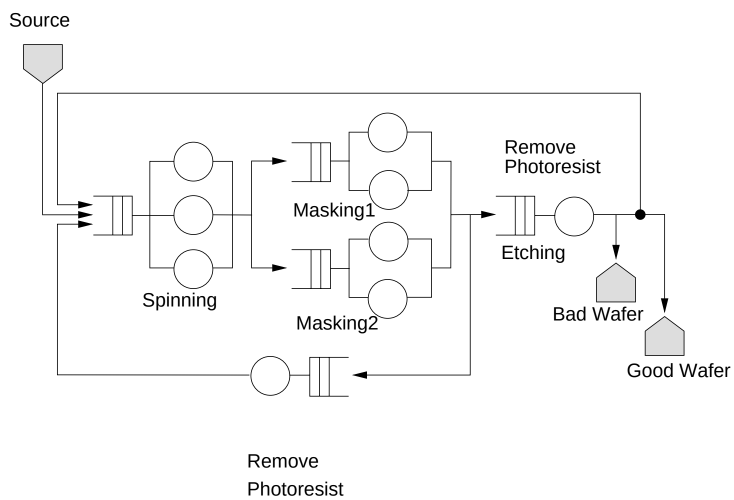
➤ Total system time and WIP (work in progress = number of work pieces in the system, waiting or in service):

m_T	WIP	$\overline{T}$
2	82.5	5.5
3	18.3	1.2

– Other numbers of machines:

m_T	m_{LA}	m_M	WIP	$\overline{T}$
3	2	3	18.3	1.22
4	2	3	17.6	1.18
3	3	3	16.9	1.12
3	2	4	15.0	1.00
3	3	4	13.6	0.90

Wafer Production System:



➤ System parameters:

m_i	p_{ij}	μ_i	λ
$m_1 = 3 \ldots 10$	$p_{12} = 0.5$	$\mu_1 = 1$	$\lambda = 1$
$m_2 = 2 \ldots 4$	$p_{13} = 0.5$	$\mu_2 = 1$	
$m_3 = 2 \ldots 4$	$p_{24} = 0.9$	$\mu_3 = 1$	
$m_4 = 2$	$p_{25} = 0.1$	$\mu_4 = 2$	
$m_5 = 1$	$p_{34} = 0.9$	$\mu_5 = 1$	
	$p_{35} = 0.1$		
	$p_{40} = 0.1$		
	$p_{46} = 0.9$		
	$p_{60} = 2/3$		
	$p_{61} = 1/3$